

# Algorithmic Game Theory

## Mechanism Design: Single Parameter Environments

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*Based on slides by Alexandros Voudouris*

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- The utility of each agent is **quasilinear in money**:
  - If agent  $i$  loses the item, then her utility is 0
  - If agent  $i$  wins the item at price  $p$ , then her utility is  $v_i - p$

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  - **Truthful auctions that maximize the social welfare**

# First-price auction

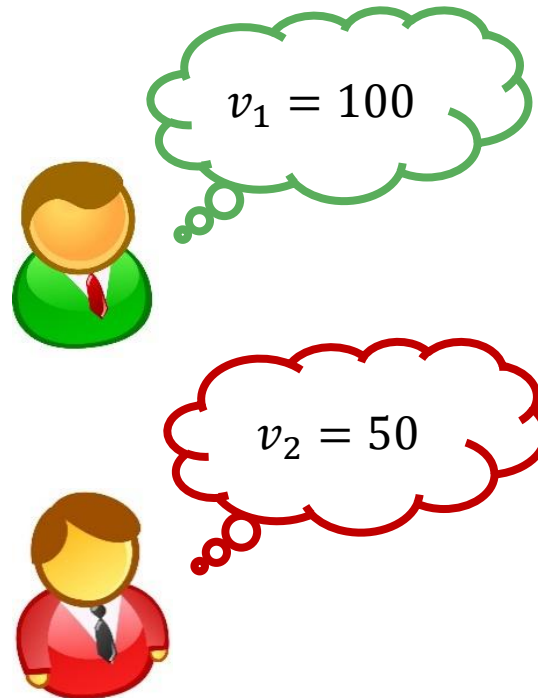
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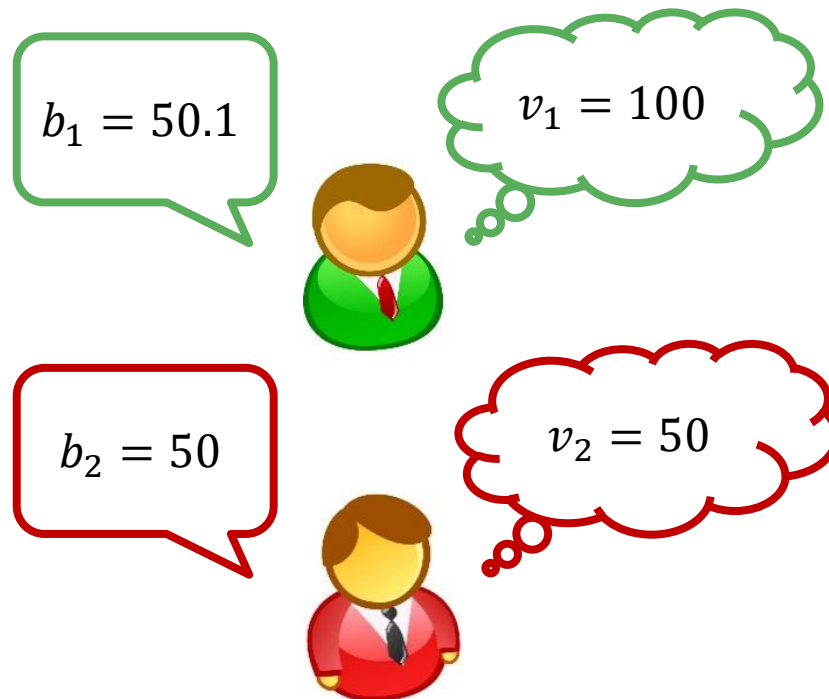
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## Theorem [Vickrey, 1961]

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- (b) is obvious:
  - the selling price is at most the winner's bid, and the bid of a truthtelling bidder is equal to her true value

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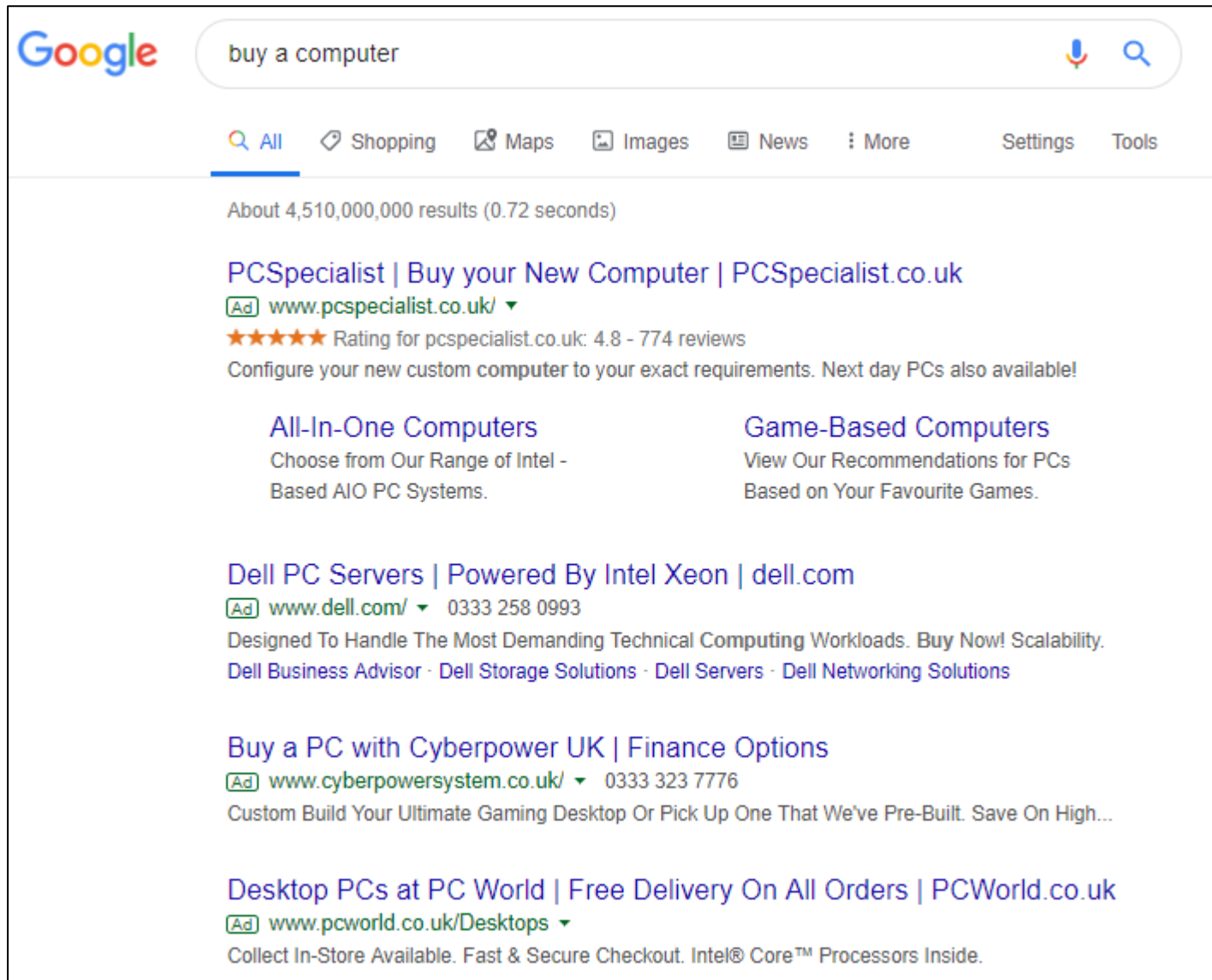
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## Case II: $v_i \geq B$

- Maximum possible utility =  $v_i - B$
- Bidder  $i$  wins the item by setting  $b_i = v_i$



# Sponsored search auctions



The image shows a Google search results page for the query "buy a computer". The search bar at the top contains the text "buy a computer" and has a microphone icon and a search icon to its right. Below the search bar, there are navigation links for "All", "Shopping", "Maps", "Images", "News", "More", "Settings", and "Tools". The search results show "About 4,510,000,000 results (0.72 seconds)".

The first sponsored advertisement is for PCSpecialist.co.uk. The headline is "PCSpecialist | Buy your New Computer | PCSpecialist.co.uk". Below the headline, there is a green "Ad" label, the URL "www.pcspecialist.co.uk/", and a star rating of 4.8 from 774 reviews. The main text of the ad says "Configure your new custom computer to your exact requirements. Next day PCs also available!". There are two sub-sections: "All-In-One Computers" with the text "Choose from Our Range of Intel - Based AIO PC Systems." and "Game-Based Computers" with the text "View Our Recommendations for PCs Based on Your Favourite Games."

The second sponsored advertisement is for Dell.com. The headline is "Dell PC Servers | Powered By Intel Xeon | dell.com". Below the headline, there is a green "Ad" label, the URL "www.dell.com/", and the phone number "0333 258 0993". The main text of the ad says "Designed To Handle The Most Demanding Technical Computing Workloads. Buy Now! Scalability. Dell Business Advisor · Dell Storage Solutions · Dell Servers · Dell Networking Solutions".

The third sponsored advertisement is for Cyberpower UK. The headline is "Buy a PC with Cyberpower UK | Finance Options". Below the headline, there is a green "Ad" label, the URL "www.cyberpowersystem.co.uk/", and the phone number "0333 323 7776". The main text of the ad says "Custom Build Your Ultimate Gaming Desktop Or Pick Up One That We've Pre-Built. Save On High...".

The fourth sponsored advertisement is for PCWorld.co.uk. The headline is "Desktop PCs at PC World | Free Delivery On All Orders | PCWorld.co.uk". Below the headline, there is a green "Ad" label, the URL "www.pcworld.co.uk/Desktops", and the text "Collect In-Store Available. Fast & Secure Checkout. Intel® Core™ Processors Inside."

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- Each bidder  $i$  has a **private value**  $v_i$  **per click**
  - Bidder  $i$  derives utility  $a_j \cdot v_i$  from slot  $j$

# Sponsored search auctions: goals

- **Truthfulness:** It is a dominant strategy for each bidder to bid her true value
- **Social welfare maximization:**  $\sum_i v_i \cdot x_i$ 
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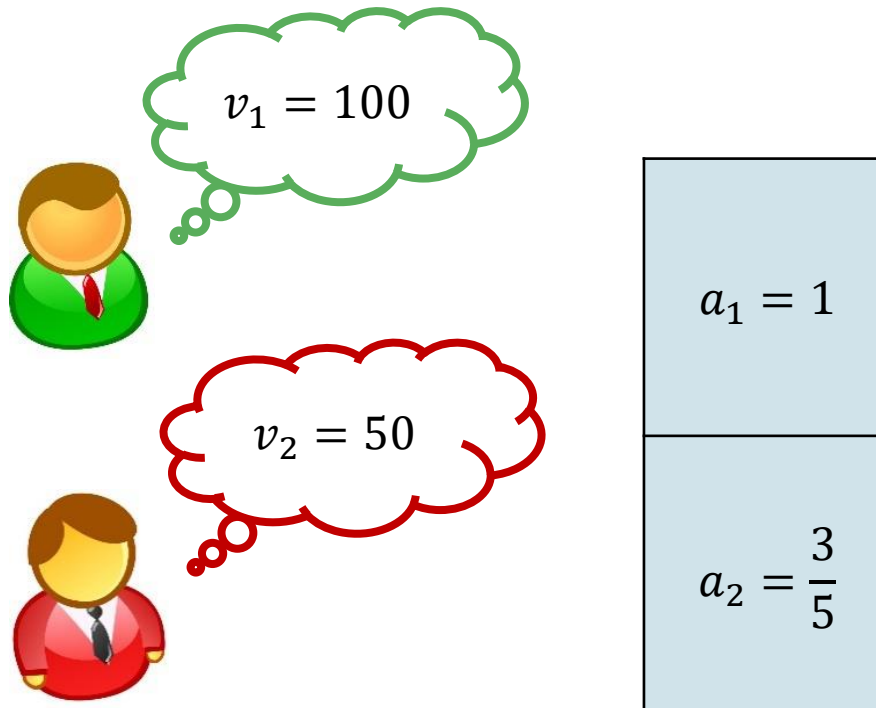
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- Can we extend the ideas we exploited for single-item auctions?

# Generalized second-price auction

- **Allocation rule:** sort the bidders in decreasing order of their bids and rename them so that  $b_1 \geq \dots \geq b_n$
- **Payment rule:** every bidder  $i \leq k$  (who is assigned at slot  $i$ ) pays the next highest bid  $b_{i+1}$  per click, and every bidder  $i > k$  pays 0

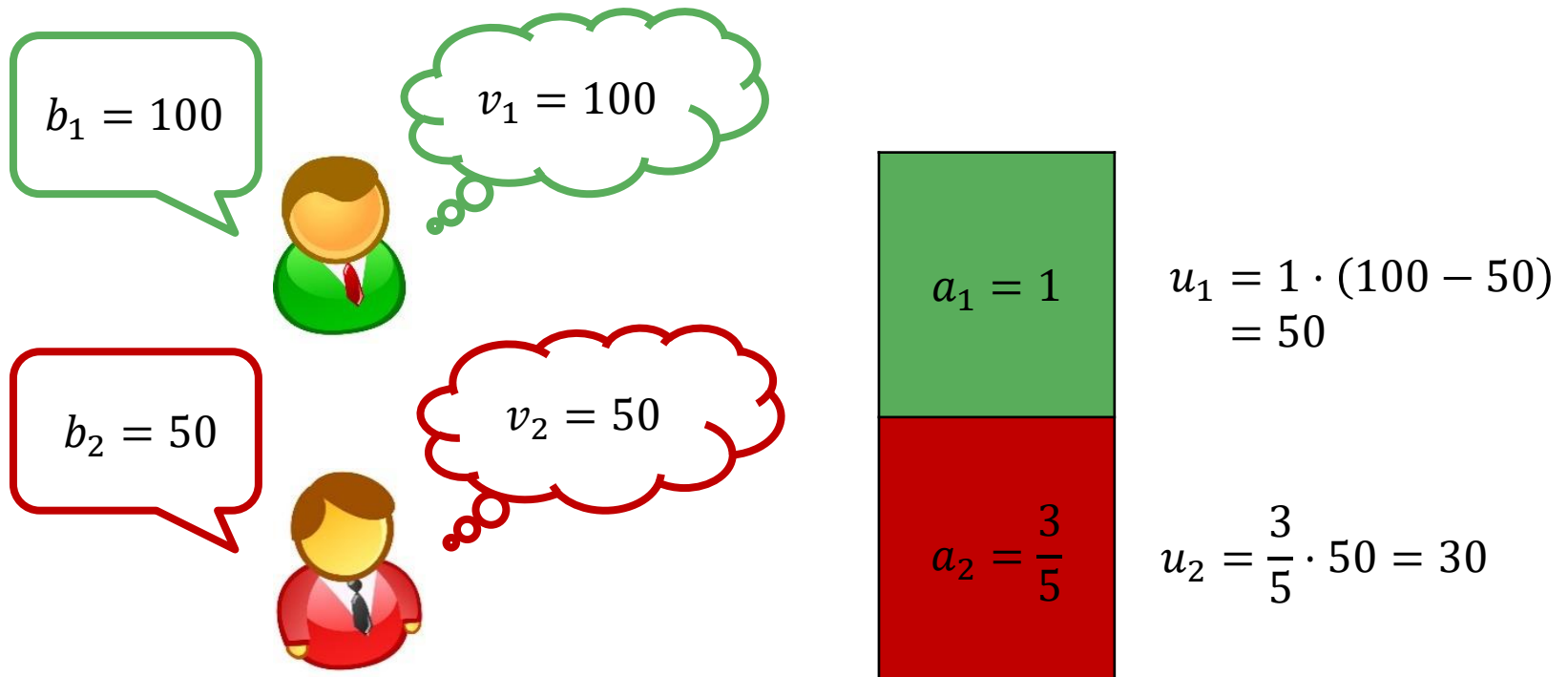
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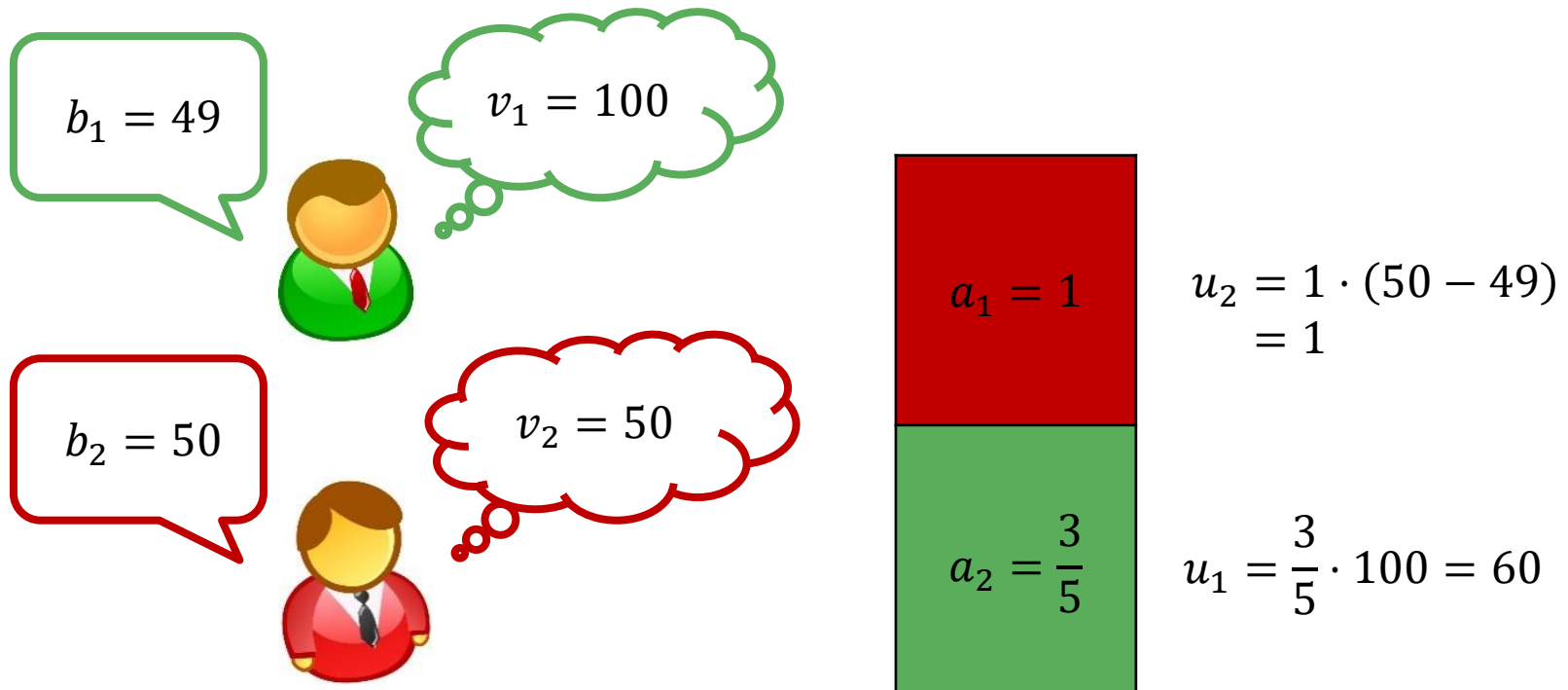
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- The utility of bidder  $i$  is  $u_i(\mathbf{b}) = v_i \cdot x_i(\mathbf{b}) - p_i(\mathbf{b})$
- Focus on payment rules such that  $p_i(\mathbf{b}) \in [0, b_i \cdot x_i(\mathbf{b})]$ 
  - $p_i(\mathbf{b}) \geq 0$  ensures that the seller does not pay the bidders
  - $p_i(\mathbf{b}) \leq b_i \cdot x_i(\mathbf{b})$  ensures non-negative utility for truthful bidders

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## Lemma [Myerson, 1981]

- (a) An allocation rule  $\mathbf{x}$  is implementable if and only if it is monotone
- (b) For every allocation rule  $\mathbf{x}$ , there exists a unique payment rule  $\mathbf{p}$  such that  $(\mathbf{x}, \mathbf{p})$  is a truthful auction



# Proof of Myerson's Lemma

- Fix a bidder  $i$ , and the bids  $\mathbf{b}_{-i}$  of the other bidders
- Given that these quantities are now fixed, we simplify our notation:
  - $x(z) = x_i(z, \mathbf{b}_{-i})$
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- The idea:
  - assuming  $(\mathbf{x}, \mathbf{p})$  is a truthful auction, the bidder has no incentive to unilaterally deviate to any other bid
  - This will give us a relation between  $\mathbf{x}$  and  $\mathbf{p}$ , which we can use to derive an explicit formula for  $\mathbf{p}$  as a function of  $\mathbf{x}$

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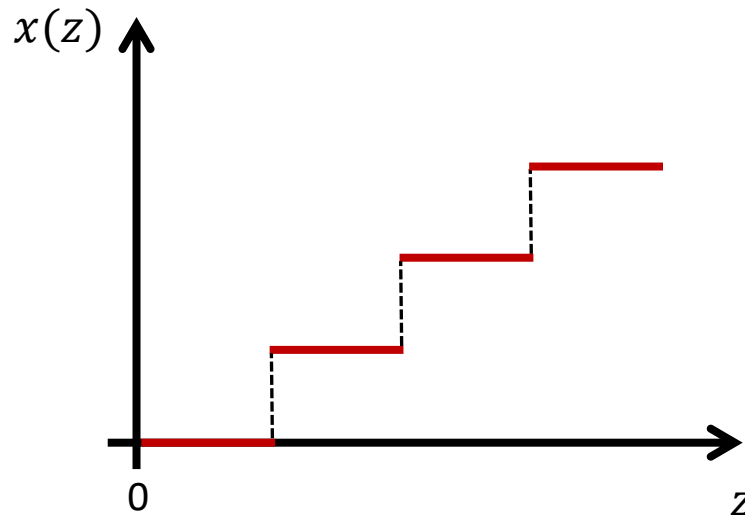
$\Rightarrow$  (a) is now proved

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- Assume  $x$  is piecewise constant, like in sponsored search auctions



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- Therefore, we can define the payment of the bidder as

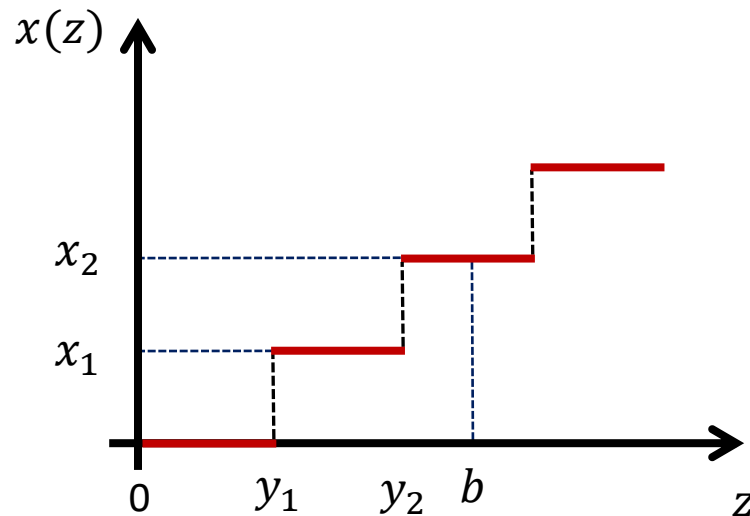
$$p(b) = \sum_{y \in [0, b]} y \cdot (\text{jump of } x \text{ at } y)$$

where  $y$  enumerates all break points of  $x$  in  $[0, b]$



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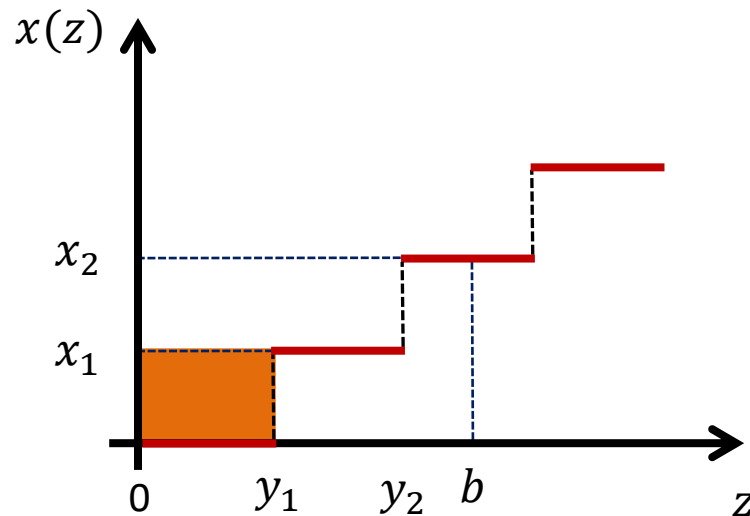
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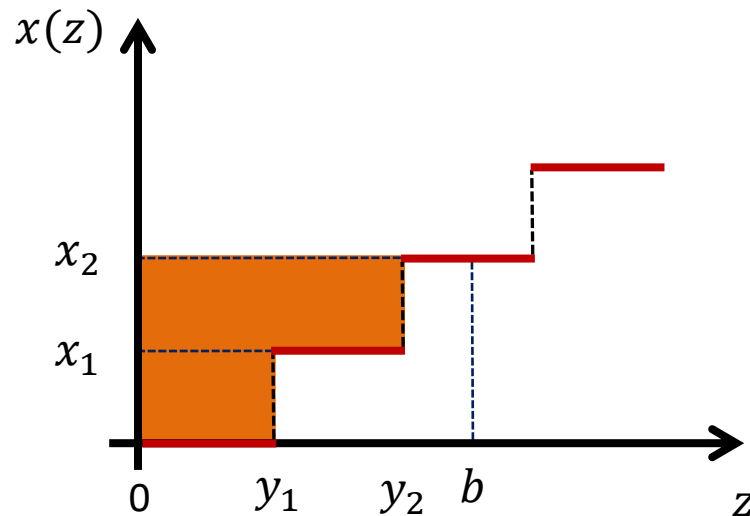
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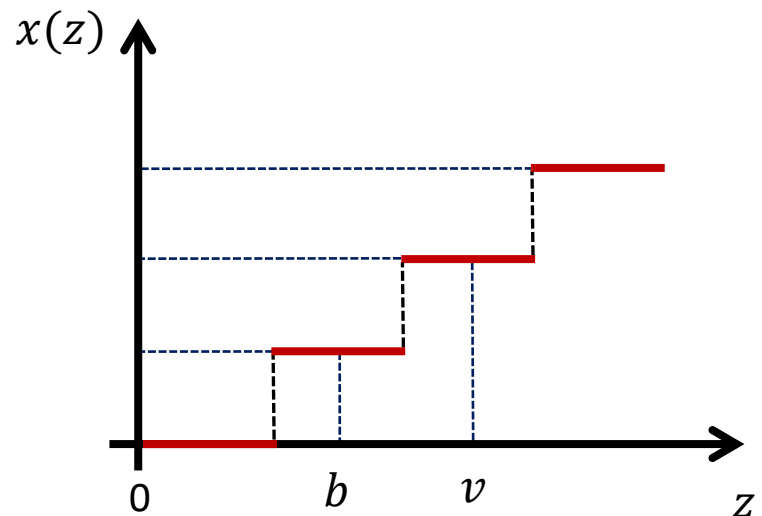
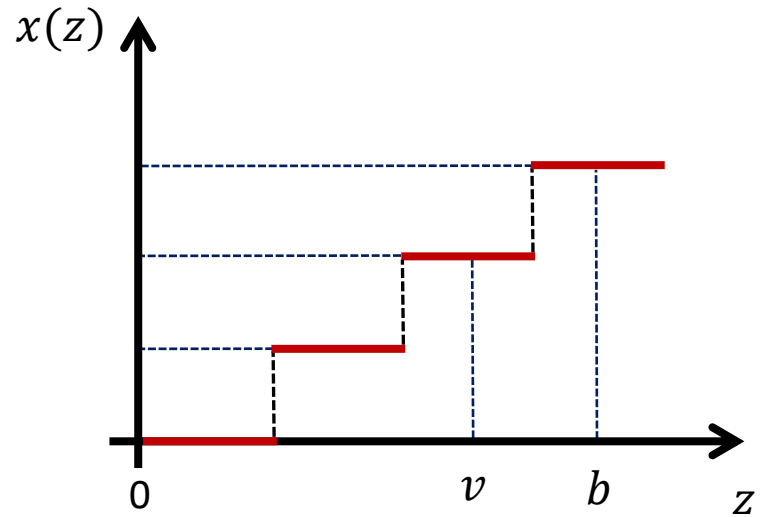
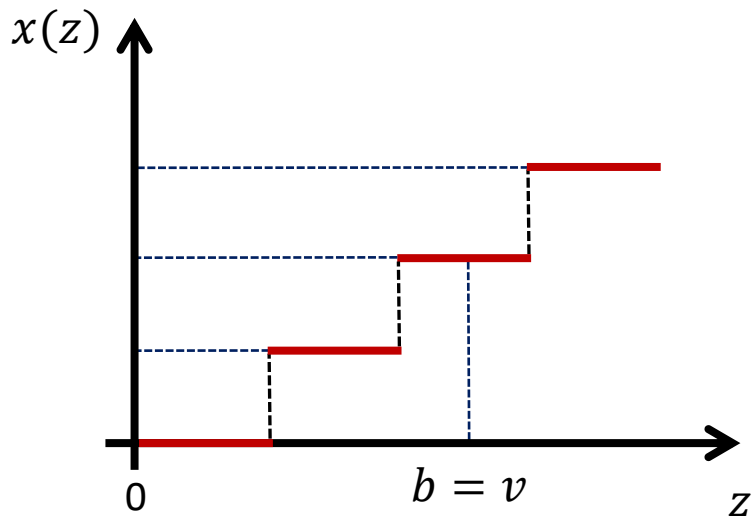
# Proof of Myerson's Lemma

- Example:

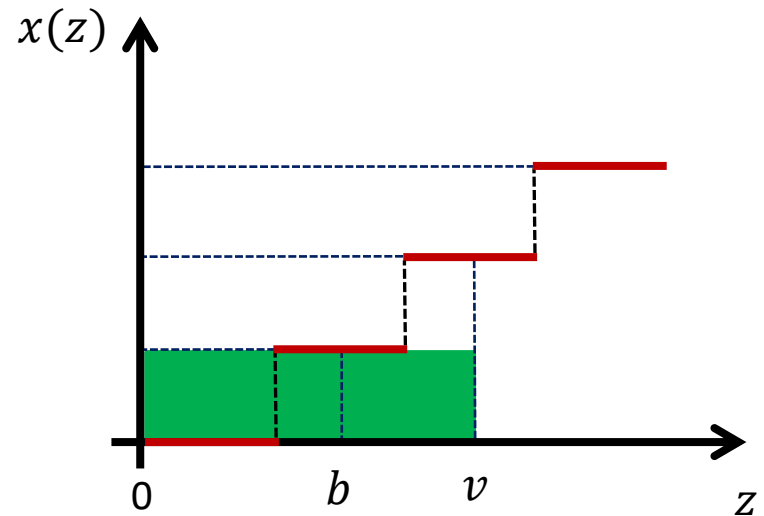
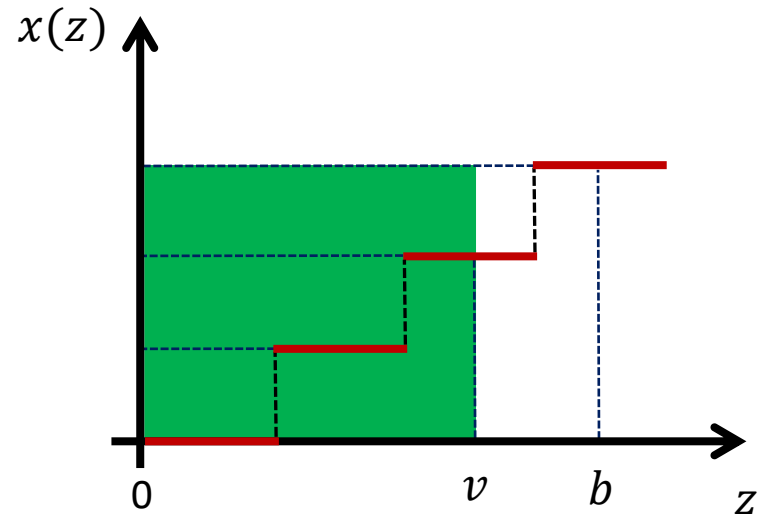
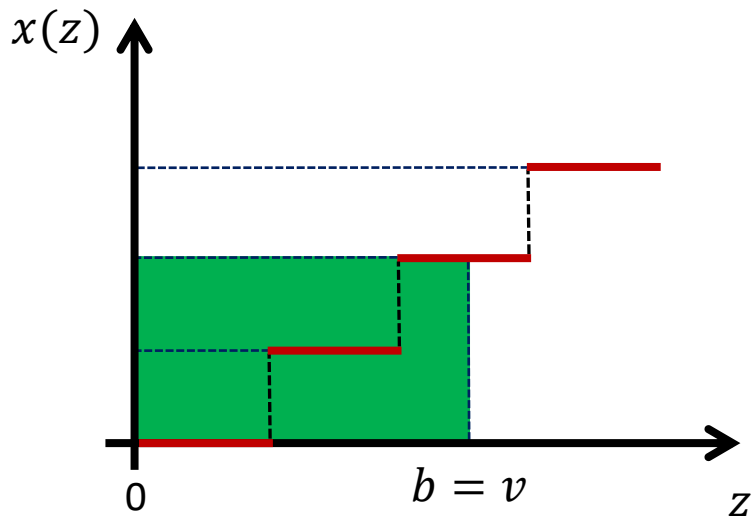


$$p(b) = \sum_{y \in [0, b]} y \cdot (\text{jump of } x \text{ at } y) = y_1 \cdot x_1 + y_2 \cdot (x_2 - x_1)$$

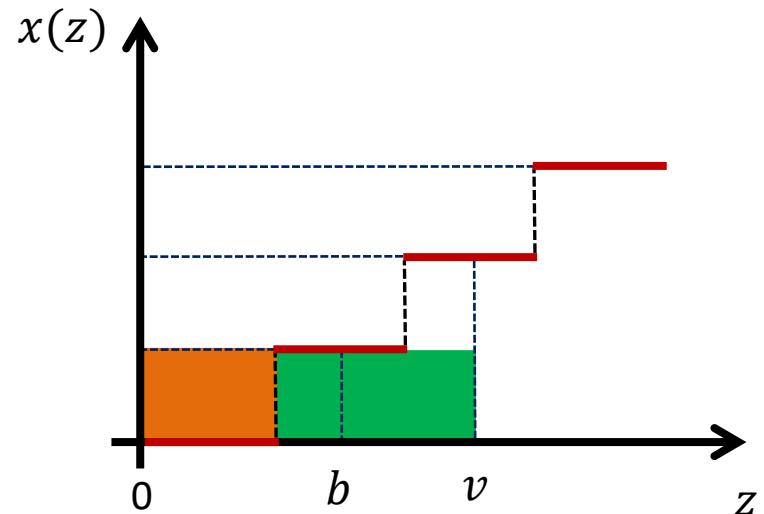
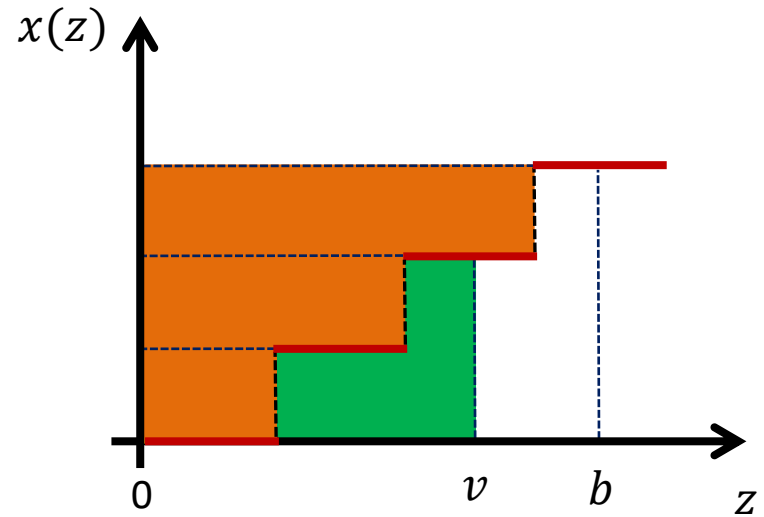
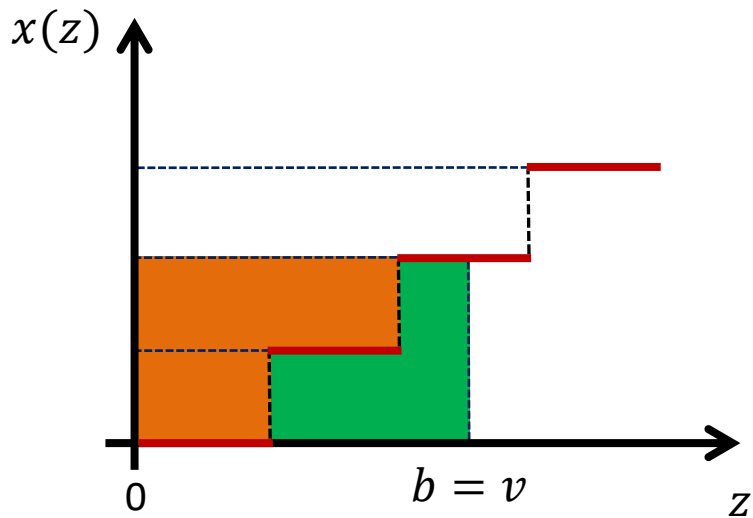
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- jump of  $x$  at  $y$ : the difference in CTR between two consecutive slots
- The total payment of the  $i$ -th highest bidder is:

$$p_i(b_i, \mathbf{b}_{-i}) = \sum_{j=i}^k b_{j+1} (a_j - a_{j+1})$$

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- **Myerson's Lemma:** a characterization of truthful mechanisms in single-parameter environments
- Using Myerson's Lemma we can design a truthful sponsored search auction